PHI454: Formal Epistemology
End Sem Exam
Introduction

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Syllabus

Darren Bradeley, Critical Introduction to Formal Epistemology

1. Bayes theorem: pp1-18
2. Problem of Induction, Grue: New problem of Induction, The paradox of Ravens (Chapter 6, 7, 8)- pp75-126
3. Old Evidence Problem: 161-168

Interactive Epistemology

- Slides
- Basic definitions: Examples, Common Knoledge

Pattern of Exam

1. True or False Questions [15]
2. Objective type questios [10]
3. Short answer Questions(upto 750-1000 words) [3 x 5= 15M]

Question 1

Bayes' theorem can only be applied when all probabilities are known with certainty.
Answer: False

Explanation

Bayes' theorem can be applied with estimated or subjective probabilities. In formal epistemology, subjective Bayesianism allows agents to assign credences that reflect their degrees of belief even when precise frequencies are unknown. It doesnt matter whether the input probabilities come from objective frequencies or subjective assessments.

Question

Bayes' theorem requires that $P(E) > 0$ for the calculation to be valid.

Answer: True

explanation

1. In practice, Bayesians ensure that the evidence in question has non-zero probability
2. The denominator in Bayes' theorem is $P(E)$, the marginal or total probability of the evidence.
3. If $P(E) = 0$, the division is undefined, making the theorem inapplicable.
4. This reflects the fact that if an event has zero probability of occurring, conditioning on its occurrence is conceptually problematic. .

Question

Bayes' theorem can be derived from the definition of conditional probability and the law of total probability.

Answer: True

Explanation

1. Starting with the definition $P(H|E) = P(H \text{ and } E)/P(E)$, and using $P(H \text{ and } E) = P(E|H) \times P(H)$, we obtain the numerator.
2. The denominator $P(E)$ can be expanded using the law of total probability: $P(E) = P(E|H) \times P(H) + P(E|\text{not-}H) \times P(\text{not-}H)$.
3. Bayes' theorem is not an independent axiom but a mathematical consequence of basic probability theory.

Question

In Bayesian epistemology, the principle of conditionalization states that the new credence in H should equal the old credence in H given E.

Answer: True

Explanation

1. The principle of conditionalization (also called probabilism) requires that when an agent learns evidence E and nothing else, their new credence function P_{new} should satisfy $P_{new}(H) = P_{old}(H|E)$. Bayes' theorem provides the mathematical mechanism for this update when the prior and likelihood are specified.
2. This principle is normative: an agent violates the laws of probability if they update in any other way after learning E with certainty

Question

Question

According to the Bayesian approach, learning that evidence E has probability zero means the hypothesis H must be false.

Answer: False

Explanation

1. If $P(E) = 0$, Bayes' theorem is undefined because we cannot condition on an event that has zero probability of occurring.
2. In practice, this situation rarely arises with real-world evidence. However, if we discover that our evidence E is impossible given our current understanding, this typically means we have made an error in assessing $P(E)$, or the evidence itself is illusory.
3. It does not directly imply H is false without further analysis.

Question

In the Bayesian framework, the prior probability $P(H)$ represents the agent's degree of belief in H before considering the new evidence E.

Answer: True

Explanation

The prior captures the agent's epistemic state regarding hypothesis H prior to the acquisition of evidence E. It reflects all previously accumulated information and background knowledge. Bayes' theorem then shows how this prior should be updated to the posterior $P(H|E)$ when new evidence E becomes available.

Question 2

The grue paradox shows that inductive reasoning is fundamentally flawed and should be abandoned.

Answer: False

Explanation

1. The grue paradox does not prove that induction is flawed. Rather, it reveals that the problem of induction is more complex than simply projecting observed patterns into the future.
2. Goodman's point is that we need a principled way to distinguish between projectable predicates (like green) and non-projectable ones (like grue).
3. The paradox challenges us to explain why we favor certain predicates over others, not to abandon induction entirely.

Question

The predicate "green" is more projectable than the predicate "grue" because it has been used more frequently in inductive inferences.

Answer: False

Explanation

1. Goodman rejects the idea that projectability is determined by past usage frequency. If we say "green" is projectable because it has been projected before, we are using an inductive argument that already assumes green is projectable.
2. This would be circular. Goodman seeks a non-circular way to distinguish projectable from non-projectable predicates, not based on historical frequency of use but on some deeper semantic or syntactic property.

Question

Goodman believed that all predicates are equally legitimate for inductive generalization.
Answer: False

Explanation

- Goodman's central insight is that not all predicates support induction equally. While "green" supports the inference that unobserved emeralds will be green, "grue" does not support the inference that unobserved emeralds will be grue (since after time t, grue emeralds would be blue).
- Goodman seeks to explain what makes some predicates projectable and others not, rather than treating all predicates as equivalent for inductive purposes.

Question

The paradox of the ravens violates the notion that confirmation should be relevant to the hypothesis being tested.

Answer: True

Explanation

1. Intuitively, observing a white shoe has nothing to do with ravens.
2. Yet Hempel's conditions of adequacy for confirmation theory imply that such observations count as confirmatory.
3. This creates a tension between formal confirmation theory and our intuitive notion that relevant evidence should concern the subject matter of the hypothesis.

Question

Hempel's conditions of adequacy for confirmation include the requirement that confirmation is transitive.

Answer: False

Explanation

1. Hempel proposed several conditions, including the equivalence condition (equivalent hypotheses are confirmed by the same evidence) and the special consequence condition (if evidence confirms H, it also confirms any hypothesis logically implied by H).
2. Transitivity is not among his conditions. In fact, transitivity of confirmation leads to absurdities. If E confirms H, and H implies G, it does not follow that E confirms G to the same degree or even at all in some cases.

Question

The problem of induction arises because deductive logic cannot bridge the gap between past observations and future predictions.

Answer: True

Explanation:

1. Deductive inferences preserve truth from premises to conclusion. If all observed ravens have been black, that does not deductively entail that the next raven will be black. The step from "All observed Fs are G" to "All Fs are G" is not deductively valid.
2. Induction requires a leap beyond what deduction can guarantee, and this leap is what Hume questioned and what Goodman's grue paradox complicates further.

Question 3

Observing a white shoe confirms the hypothesis that all ravens are black.
Answer: True

Explanation

1. if a piece of evidence confirms a hypothesis, it also confirms any logically equivalent hypothesis. All ravens are black is logically equivalent to All non-black things are non-ravens.
2. A white shoe is non-black and non-raven, so it satisfies the equivalent formulation and therefore confirms the original hypothesis.

Question 4:

The principle of uniformity of nature is itself known through inductive reasoning.
Answer: True

Explanation

1. Humes fork: deduction or matters of fact. Hume argued that our belief in the uniformity of nature cannot be proven deductively or through pure reason. It arises from our experience of the past regularity of nature.
2. induction is justified by the uniformity principle, which is justified by induction.

Question 5

In Bayesian terms, evidence E disconfirms hypothesis H when $P(H|E) < P(H)$.
Answer: True

Explanation

1. When the posterior probability $P(H|E)$ is less than the prior probability $P(H)$, the evidence has reduced the probability of the hypothesis.
2. This is the Bayesian definition of disconfirmation. The evidence shifts the agent's credence away from the hypothesis, making it less likely in light of the new information.

MCQ

Instructions: Select the best answer for each question. The correct answer is indicated after each question (in your answersheet).

MCQ Question

Question 1

What is the correct formulation of Bayes' theorem for computing $P(H|E)$, the probability of hypothesis H given evidence E?

A) $P(H|E) = P(E|H) \times P(H)/P(E)$
B) $P(H|E) = P(E|H) + P(H) - P(E)$
C) $P(H|E) = P(E|H) \times P(E)/P(H)$
D) $P(H|E) = P(E|H) \times P(H) \times P(E)$

Correct Answer: A

Bayes' theorem states $P(H|E) = P(E|H) \times P(H)/P(E)$. This formula updates the prior probability $P(H)$ to the posterior $P(H|E)$ using the likelihood and marginal evidence probability.

MCQ Question

Question

A medical test for a disease has a 95 percent true positive rate and a 5 percent false positive rate. If 1 percent of the population has the disease, what is the approximate probability that a person actually has the disease given a positive test result?

A) 95 percent
B) 16 percent
C) 50 percent
D) 5 percent

Correct Answer: B

Explanation

Using Bayes:
$P(Disease|Positive) = 0.95 \times 0.01/(0.95 \times 0.01 + 0.05 \times 0.95) = 0.0095/0.059 = 0.161$ or 16.0 percent.

Question

MCQ

According to Carl Hempel's ravens paradox, which statement is equivalent to "All ravens are black"?

A) All non-black things are non-ravens
B) All black things are ravens
C) Some ravens are black
D) No non-black things are ravens

Correct Answer: A

explanation

By contrapositive, "All ravens are black" is equivalent to "All non-black things are non-ravens." This logical equivalence drives Hempel's paradox.

MCQ

MCQ4

Nelson Goodman's grue paradox challenges the problem of induction by introducing the predicate "grue," which applies to things examined before time t if they are green, and to things not examined before time t if they are blue. What is the main problem this creates?

A) It shows that all inductive inferences are invalid

B) It demonstrates that any instance of a general law confirms both that law and its competitor

C) It proves that color predicates are meaningless

D) It establishes that induction is superior to deduction

Correct Answer: B

Explanation

The grue definition makes "All emeralds are green" and "All emeralds are grue" both confirmed by pre-time-t emeralds, yet they make opposite predictions. This shows the problem of distinguishing projectable predicates.

MCQ5

David Hume argued that our reliance on inductive reasoning depends on which of the following?

A) A priori logical necessity

B) The principle of uniformity of nature

C) Divine revelation

D) Mathematical demonstration

Correct Answer: B

explanation

Hume argued that induction presupposes nature is uniform, but this principle cannot be proven deductively or through reason. It arises from custom and experience.

MCQ6

In the context of Bayes' theorem, what does the likelihood $P(E|H)$ represent?

A) The prior probability of the hypothesis

B) The probability of the evidence given the hypothesis is true

C) The probability of the hypothesis given the evidence

D) The marginal probability of the evidence

Correct Answer: B

Explanation

The likelihood $P(E|H)$ is the probability of observing evidence E given that hypothesis H is true. It measures how well the hypothesis predicts the evidence

MCQ

In Bayesian confirmation theory, when does evidence E confirm hypothesis H?

A) When $P(H|E) > P(H)$

B) When $P(H|E) = P(H)$

C) When $P(H|E) < P(H)$

D) When $P(E) = 1$

Correct Answer: A

Explanation

In Bayesian confirmation theory, evidence confirms a hypothesis when it raises the probability of that hypothesis. $P(H|E) > P(H)$ is the standard definition of confirmation.

Question- 5M

A doctor has two tests for a rare disease. Test A has a 99 percent true positive rate and 1 percent false positive rate. Test B has a 90 percent true positive rate and 0.1 percent false positive rate. The disease prevalence is 0.1 percent. Using Bayes' theorem, calculate and compare the posterior probabilities of having the disease given a positive result on each test. Which test provides stronger confirmation of the disease, and why might a doctor choose the test with the lower true positive rate?

Answer

1. For Test A:
$P(D|A+) = 0.99 \times 0.001/(0.99 \times 0.001 + 0.01 \times 0.999) = 0.00099/0.01098 = 0.090$ or 9.0 percent.
2. For Test B:
$P(D|B+) = 0.90 \times 0.001/(0.90 \times 0.001 + 0.001 \times 0.999) = 0.0009/0.001899 = 0.474$ or 47.4 percent.
3. Test B provides much stronger confirmation despite the lower true positive rate because its false positive rate is dramatically lower.
4. A doctor might choose Test B when the cost of a false positive (unnecessary treatment, anxiety) is high, or when the disease is rare enough that false positives dominate. This illustrates that likelihood ratios, not individual test characteristics, determine confirmatory power.

Q2

Hume's problem of induction leads to a circular justification: we use inductive reasoning to establish the uniformity of nature, which then justifies induction. Explore whether this circularity is epistemically vicious or whether it might be an unavoidable feature of any knowledge system. Compare three responses: (1) the pragmatic response (induction works, so use it), (2) the naturalized epistemology response (induction is evolved and reliable), and (3) the Kantian response (uniformity is a necessary condition of experience). Which offers the most satisfactory account and why?

Answer: clue

Strong answers will:

1. Clearly articulate the circularity argument
2. Explain why the pragmatic response is unsatisfying (it abandons justification for utility)
3. Show how naturalized epistemology makes circularity acceptable (reliability is causal, not logical)
4. Evaluate the Kantian response (uniformity as transcendental condition)
5. Argue for one response with clear dialectical reasoning

q3

Goodman's grue predicate is defined as: x is grue if x is examined before time t and is green, or x is not examined before time t and is blue. Suppose all emeralds examined before time t are green. Show how this same evidence equally supports both "All emeralds are green" and "All emeralds are grue." Explain why this creates a problem for any theory of induction that treats all generalizations the same way.

Answer

1. Let E = "All examined emeralds before time t are green." Let G = "All emeralds are green." Let R = "All emeralds are grue."
2. For any examined emerald e before time t: if e is green, then e is grue (by definition of grue). So E confirms G directly (instances of green emeralds). But E also confirms R, because every examined emerald satisfying "green" also satisfies "grue" before time t.
3. Since both G and R are equally confirmed by E yet make opposite predictions after time t (G predicts green, R predicts blue), we cannot use the evidence to decide between them. This shows that any inductive rule that treats all predicates symmetrically cannot distinguish good inductions from bad ones.
4. We need an independent account of projectability.

Q4

Synthesize the lessons of the problem of induction, the grue paradox, and the paradox of ravens into a single framework for thinking about the ethics of belief. If our belief-forming practices (induction, confirmation, projection) rest on contested philosophical foundations, what obligations do we have when forming beliefs about matters of public policy, scientific research, or personal relationships? Construct an argument for epistemic humility in the face of Humean skepticism, while also defending the pragmatic necessity of inductive reasoning in daily life.

Answer

evaluation criteria

1. Synthesize Hume (induction lacks foundation), Goodman (projectability problem), Hempel (con- firmation paradox)
2. Construct a framework linking epistemic humility to these paradoxes
3. Apply to public policy (climate change, vaccination) where inductive projections guide action
4. Argue that we must act on induction while acknowledging its limitations
5. Defend a balanced position: pragmatic necessity plus epistemic modesty

Question5

Q5

The paradox of the ravens has been addressed through Bayesian methods, relevance theories, and logical approaches. Compare these three strategies. The Bayesian approach shows that non-black non-ravens provide mathematically valid but vanishingly small confirmation. The relevance approach argues that only evidence directly concerning ravens counts as relevant confirmation. The logical approach modifies the conditions of adequacy for confirmation to block the equivalence condition. Which approach best captures our actual epistemic practices, and what does this reveal about the relationship between formal epistemology and common sense?

Answer

1. Explain the Bayesian result (confirmation present but negligible for non-ravens)
2. Articulate the relevance approach (only direct evidence counts)
3. Describe logical modifications (rejecting the equivalence condition)
4. Compare to actual scientific practice (scientists care about relevant tests)
5. Conclude that formal epistemology needs to incorporate relevance, not just logic.